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Predictive active control of MDOF structures

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SUMMARY

This paper presents a theoretical study of a predictive active control system used to improve the response of multi-degree-of-freedom (MDOF) structures to earthquakes. As an example a building frame equipped with electrorheological (ER) dampers is considered. The aim of the design is to find a combination of forces that are produced by the ER dampers in order to obtain an optimal structural response. The mechanical response of ER fluid dampers is regulated by an electric field. Linear auto-regressive model with exogenous input (ARX) is used to predict the displacements and the velocities of the frame in order to overcome the time-delay problem in the control system. The control forces in the ER devices are calculated at every time step by the optimal control theory (OCT) according to the values of the displacements and of the velocities that are predicted at the next time step at each storey of the structure.

A numerical analysis of a seven-storey ER damped structure is presented as an example. It shows a significant improvement of the structural response when the predictive active control system is applied compared to that of an uncontrolled structure or that of a structure with controlled damping forces with time delay. The structure's displacements and velocities that were used to obtain the optimal control forces were predicted according to an 'occurring' earthquake by the ARX model (predictive control). The response was similar to that of the structure with control forces that were calculated from a 'known' complete history of the earthquake's displacement and velocity values, and were applied without delay (instantaneous control). Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: active control; predictive control; electrorheological dampers

INTRODUCTION

Passive energy dissipating systems have been installed in existing buildings resulting in improved structural response to earthquakes. However, such systems have inherent limitations. For example, they are generally tuned to the first vibration mode, while active dampers can be effective over a much wider frequency range.

Electrorheological (ER) dampers are known as effective devices for vibration control of mechanical systems [1]. The resulting damping force developed by an ER device depends on its

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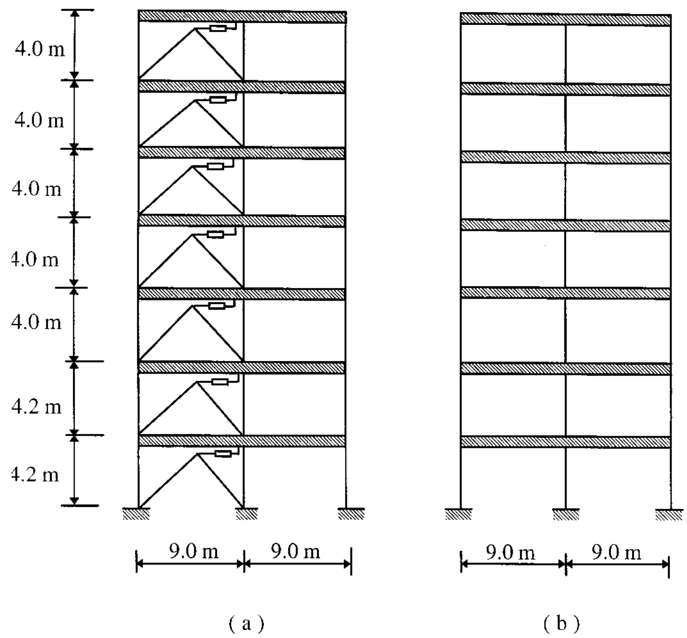

Figure 1. Seven-storey structure (a) ER damped, (b) uncontrolled.

fluid physical properties, on the pattern of flow in the damper and on its size. When an electric field is applied, the behaviour of the electrorheological fluid is nearly viscoplastic and the shear stress in it has to exceed the developed 'yield' stress in order to initiate flow. Thus, the controllable viscoplastic behaviour of ER dampers is facilitated.

The magnitude of the forces that can be developed in the ER dampers are considerably large, making them suitable for structural engineering applications. However, a time delay occurs in the control system, because the control forces can be applied only after they are calculated, and not instantaneously. This problem may be solved by using a predictive control system. At each time step a linear model is used to obtain the structure's displacements and velocities at the next time step (the output of the identified system) as a function of the known ground accelerations' history up to the current time step (the input of the identified system). Thus, the control forces are calculated according to the predicted values of the displacements and velocities at the next time step (rather than at their occurring values). In practice, they are applied by varying the electric field at each time step according to the forces that are calculated for the displacements and velocities, which are expected to occur after a certain time delay.

Actively controlled ER dampers are commonly placed between chevron braces and the rigid upper floor diaphragm of each storey of the structure (Figure 1(a)). The optimal control forces are determined by using the instantaneous active control theory (ACT), [2], as described by Ribakov and Gluck [3].

This paper describes the application of the predicted displacements and velocities in an optimization procedure in order to obtain the optimal control forces of an ER damped active control system.

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INSTANTANEOUS CONTROL

The response of a structure provided with supplemental dissipating devices is described by the following dynamic equation of equilibrium [2]:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Lf_{\mathrm{e}}(t) + Du(t) \tag{1}$$

where M, C and K are the mass, damping, and stiffness matrices, respectively; $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$ are the displacement, velocity and acceleration vectors, respectively, $u(t)$ is the vector of forces in the supplemental devices, $f_{\mathrm{e}}(t)$ is the external excitation, D and L are the control and excitation-forces location matrices, respectively.

The resulting damping force in an ER device [1] is

$$u_{\mathrm{ER}}(t) = C_{\mathrm{d}}\dot{x}(t) + F\,\mathrm{sign}[\dot{x}(t)] \tag{2}$$

where F is the controllable yield force, C_{d} is the viscous characteristic of the device. The viscous characteristics of the ER devices can be determined by application of the passive control theory (PCT) presented by Gluck *et al.* [4].

The system of the differential equations (1) is simplified by transformation into the space-state form:

$$\dot{z}(t) = Az(t) + Bu(t) + Hf_{\mathrm{e}}(t) \tag{3}$$

where $z(t) = [x(t), \dot{x}(t)]^{\mathrm{T}}$ is the $2n$ state-space vector of the displacements and velocities of the structure, and A is the system matrix given by

$$A_{2n\times 2n} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{4}$$

and B and H are the location matrices specifying, respectively, the locations of controllers and external excitations in the state space:

$$B_{2n\times 2n} = \begin{bmatrix} 0 \\ M^{-1}D \end{bmatrix} \quad \text{and} \quad H_{2n\times r} = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \tag{5}$$

Application of active controlled devices provides a more truly optimal control compared to passive dampers because the excitation term is not ignored in the derivation of the Riccati matrix [2]. The external excitation history is available up to the time instant at which it occurs and it is utilized to improve the response of the structure.

Considering a closed-loop instantaneous control system [2] the control forces vector at each time step, t, is obtained as follows:

$$u(t) = -\frac{\Delta t}{2} R^{-1} B^{\mathrm{T}} Q z(t) \tag{6}$$

where Δt is the time increment, and R and Q are weighting matrices, whose magnitudes are assigned according to the relative importance attached to the state variables and to the control forces [2]:

$$Q = I \tag{7}$$

$$R = 10^{-m} I \tag{8}$$

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I is a $2n \times 2n$ unit diagonal matrix, and m is a parameter which is used to tune the systems within a practical range.

The response state vector is

$$z(t) = \left[I + \frac{\Delta r^2}{4} BR^{-1} B^{\mathrm{T}} Q \right]^{-1} \left[T d(t - \Delta t) + \frac{\Delta t}{2} H f_{\mathrm{e}}(t) \right] \tag{9}$$

where

$$d(t - \Delta t) = \exp(\Lambda \Delta t) T^{-1} \left\{ z(t - \Delta t) + \frac{\Delta t}{2} [B u_{\mathrm{c}}(t - \Delta t) + H f_{\mathrm{e}}(t - \Delta t)] \right\} \tag{10}$$

and T is a $2n \times 2n$ modal matrix whose columns are eigenvectors of A, and Λ is a diagonal matrix whose diagonal elements are the complex eigenvalues of the matrix A:

$$\Lambda = T^{-1} A T \tag{11}$$

PREDICTIVE CONTROL

Instantaneous application of the optimal forces yields significant improvement in the structure's behaviour [3]. However, in practice, there is a delay from the time of the forces calculation until they are applied. According to the predictive control approach which is presented here, the structure's response (displacements and velocities) is evaluated at the time when the control forces are expected to be applied. Thus, the control forces are calculated according to their actual application time.

The main problem in a system identification is to find a suitable mathematical model which describes its dynamic behaviour. Only after the parameters of such a model are estimated it is possible to utilize prior knowledge of the system's response in order to to predict its future behaviour.

A large number of algorithms to estimate parameteric models for linear systems are available [5]. An output of each model coincides with the real system's response when the number of input signals increases to infinity. However, a minimal number of parameters, and a small error (bias) and uncertainties of the estimates are important requirements of a good model [6].

In this study a black box linear model, without any available physical knowledge of the future earthquake history, is used to obtain the relationship between the input of the system (ground accelerations) and its output (displacements and velocities at each storey of the structural frame).

Assuming that the input signal, $f_{\mathrm{e}}(t)$, and the output signal, $z(t)$, are related by a linear system, the relationship can be written as follows [7]:

$$z(t) = G(q) f_{\mathrm{e}}(t) + v(t) \tag{12}$$

where $G(q)$ is the 'transfer function' of the system, and $v(t)$ is an additional disturbance.

$$G(q) = \sum_{k=1}^{\infty} g(k) q^{-k} \tag{13}$$

$$G(q) f_{\mathrm{e}}(t) = \sum_{k=1}^{\infty} g(k) f_{\mathrm{e}}(t - k\Delta t) \tag{14}$$

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q and q^{-1} denote, respectively, shift and delay operators, where

$$q^{-1} f_e(t) = f_e(t - \Delta t) \tag{15}$$

The numbers $\{g(k)\}$ in Equation (14) are called the impulse response of the system at time step k.

A commonly used linear black box parametric model is the ARX model with exogenous input [5]:

$$\tilde{A}(q) z(t) = \tilde{B}(q) f_e(t - n_k \Delta t) + e(t) \tag{16}$$

where $\tilde{A}$ includes the auto-regressive parameters, $\tilde{B}$ is the exogenous part, and the time delay from the input to output is equal to $n_k \Delta t$. $\tilde{A}$ and $\tilde{B}$ are polynomials in the delay operator q^{-1}:

$$\tilde{A}(q) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-n_a} \tag{17}$$

$$\tilde{B}(q) = b_1 + b_2 q^{-1} + \cdots + b_{n_b} q^{-n_b + 1} \tag{18}$$

Here n_a and n_b are the orders of the polynomials.

Application of the shift operator, q, [Equation (15)] with $z(t)$ [Equation (12)] with the polynomials $\tilde{A}(q)$ and $\tilde{B}(q)$ [Equations (17) and (18)] leads to:

$$\tilde{A}(q) z(t) = z(t) + a_1 z(t - \Delta t) + \cdots + a_{n_a} z(t - n_a \Delta t) \tag{19}$$

$$\tilde{B}(q) f_e(t - n_k \Delta t) = b_1 f_e(t - n_k \Delta t) + b_2 f_e [t - (n_k + 1)\Delta t] + \cdots + b_{n_b} f_e [t - (n_k + n_b - 1)\Delta t] \tag{20}$$

Equation (16) can be now rewritten in an explicit form as follows:

$$\begin{aligned} &z(t) + a_1 z(t - \Delta t) + \cdots + a_{n_a} z(t - n_a \Delta t) \\ &\quad = b_1 f_e(t - n_k \Delta t) + b_2 f_e [t - (n_k + 1)\Delta t] + \cdots + b_{n_b} f_e [t - (n_k + n_b - 1)\Delta t] + e(t) \end{aligned} \tag{21}$$

By analysing the building's response at discrete-time increments during the earthquake, the dampers' optimal forces at every structural level are calculated according to the predicted values of the displacements and velocities. The electric field in each device is varied in such a way that it produces at time, t, the forces that were obtained by the optimization procedure at time $(t - n_k \Delta t)$ (according to the predicted displacements at time t). If the optimal force is less than the viscous part then no electric field is needed. An optimal force that is greater than the viscous part, requires that the electric field will produce in the ER device a supplemental force that is equal to the difference between the optimal value and the viscous part.

NUMERICAL EXAMPLE

In order to investigate the effectiveness of the proposed method, simulations were carried out of a seven storey, shear framed structure, with stiff beams (Figure 1(b)). The response was computed for four different seismic excitations: El-Centro S00E (1940), Taft N21E (1952), Loma-Prieta N90E (1989), and Eilat EL1226NS (1995). All simulations were performed using routines written in MATLAB [8].

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The structure was characterized by the following matrices:

$$M = 8.75 \times 10^4 \, I_{7\times 7} \text{ (kg mass)}$$

$$K = \begin{bmatrix} 29.28 & -14.64 & & & & & 0 \\ -14.64 & 31.59 & -16.95 & & & & \\ & -16.95 & 30.96 & -1401 & & & \\ & & -14.01 & 28.02 & -14.01 & & \\ & & & -14.01 & 25.13 & -11.12 & \\ & & & & -11.12 & 22.24 & -11.12 \\ 0 & & & & & -11.12 & 11.12 \end{bmatrix} \times 10^7 \text{ (N/m)}$$

where M is the mass matrix of the structure, I is a unit diagonal matrix, and K is the structure's stiffness matrix.

An initial damping ratio of 1% was assumed for the uncontrolled structure's first vibration mode. The PCT implemented in a MATLAB routine was used to obtain the optimal viscous characteristics of the devices at each level of the structure.

At each time step of an occurring earthquake the 'MATLAB System Identification Toolbox' [7] was used to calculate the ARX model in order to predict the structure's displacements and velocities at the next time step. These predicted values were used to obtain the optimal control forces in the dampers according to the ACT.

Numerical analyses of the structure were performed for the following four cases: an uncontrolled structure, a structure with an ideal instantaneous control system (without a time delay), a structure with a realistic control system with a time delay, and a structure with a predictive control system. Note that the instantaneous controlled structure is of theoretical importance and it was used for the purpose of comparison with the predictive controlled structure.

The response of the active controlled structure with prediction and without it is similar under the Taft and the Eilat earthquake (Figures 2(a) and 2(b)). However, Figures 2(c) and 2(d) show that under different earthquake histories a damping system with time delay may become hazardous even compared to that of an uncontrolled structure. Application of the predictive controlled system improved the response in all cases.

Peak displacements and base shear forces of the uncontrolled structure, of the ER damped structure with an instantaneous control system, and of an ER damped structure with a predictive control system are presented in Tables I, II and III, respectively. Time histories of the roof displacements, of the roof accelerations and of the base shear forces for these cases are shown in Figures 3, 4 and 5, respectively.

Reductions of up to 65 per cent were obtained in the peak displacements and peak accelerations of the structure with the predictive control system (see Tables I, III, and Figures 3 and 4). There were no significant increase of the peak base shear forces of the structure with the actively controlled devices compared to those of the uncontrolled structure. The structure's response to the considered earthquakes was similar with both the predictive control system and the ideal instantaneous control system.

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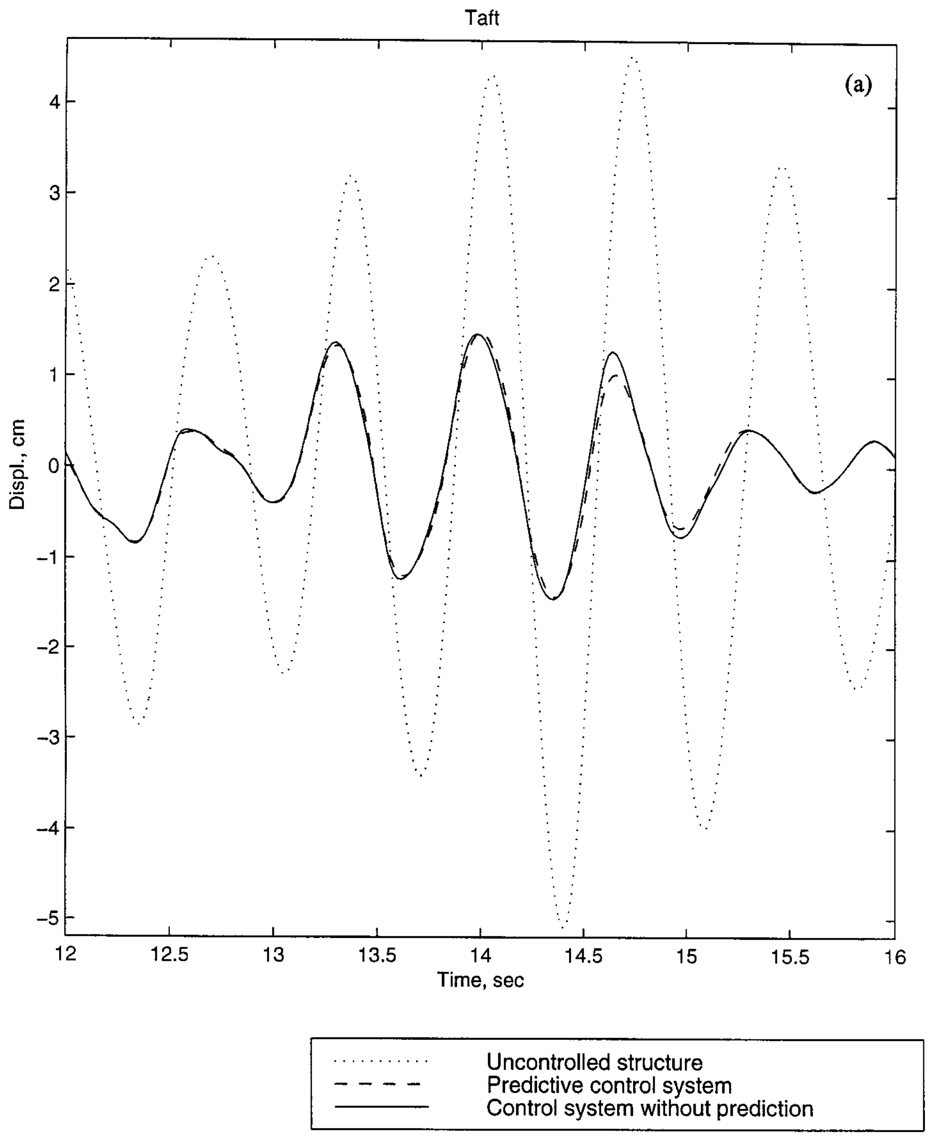

Figure 2. Roof displacement histories of the uncontrolled structure and of the active controlled structure with prediction and without it under: (a) Taft, (b) Eilat, (c) El-Centro, (d) Loma-Prieta earthquakes.

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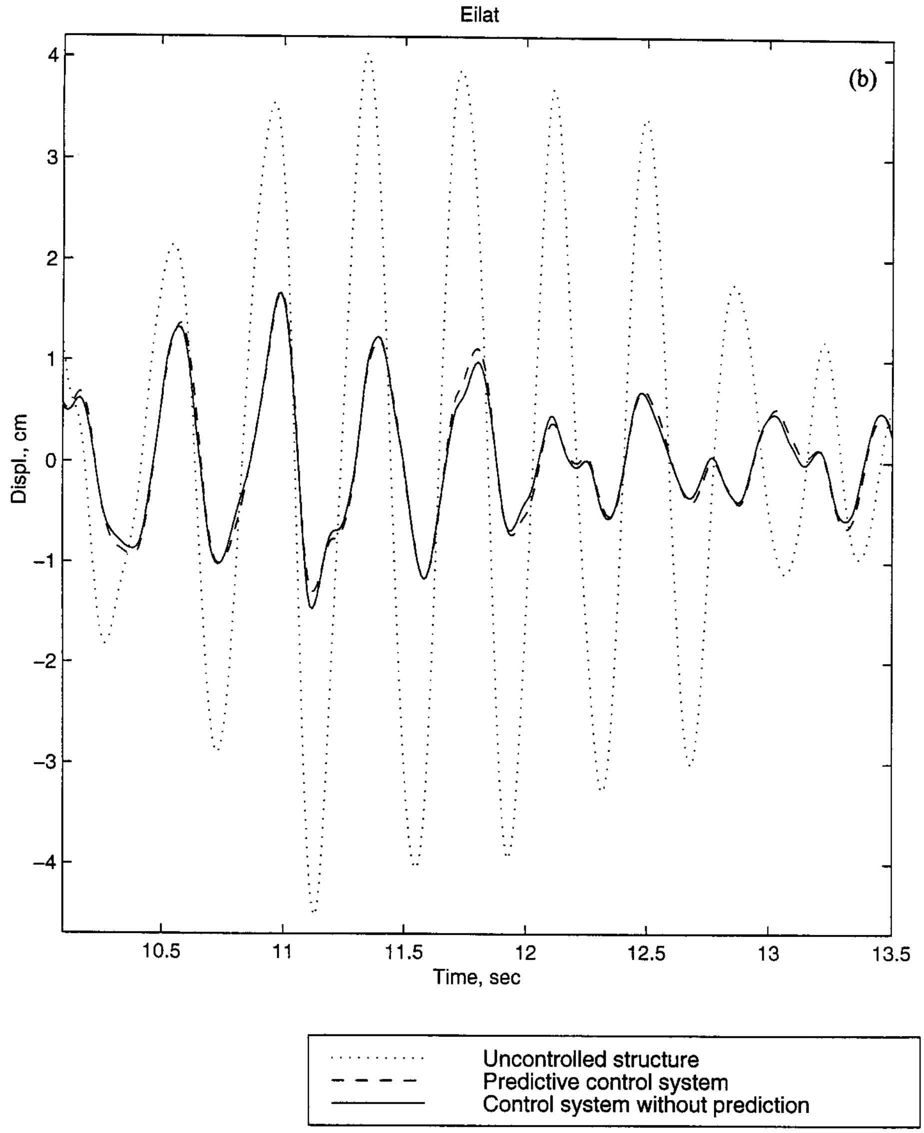

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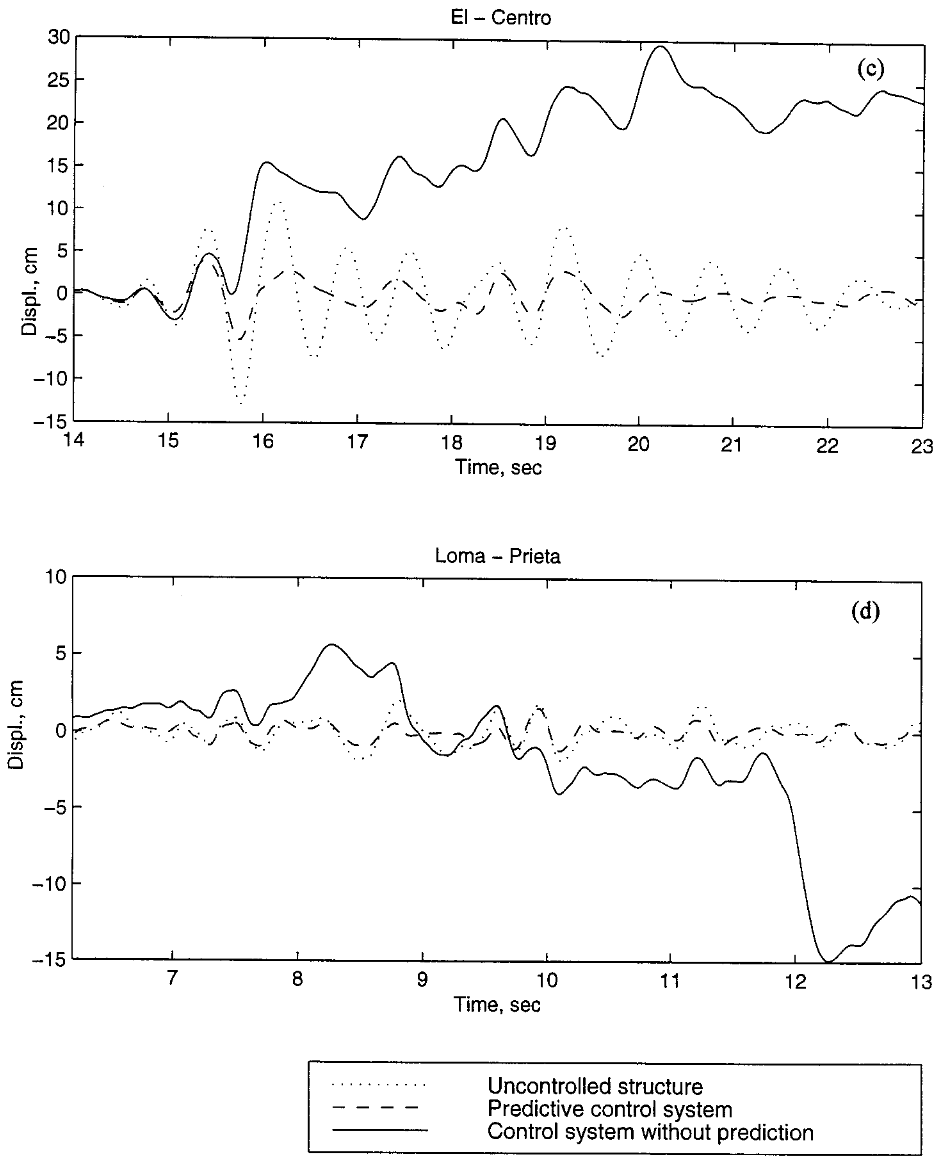

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Table I. Response of the uncontrolled structure to different earthquakes.

	Storey number	El-Centro	Taft	Loma-Prieta	Eilat
Displacement (cm)	7	12.85	5.10	2.08	4.52
	6	12.07	4.79	1.88	4.23
	5	10.56	4.19	1.57	3.69
	4	8.83	3.52	1.28	3.07
	3	6.72	2.70	1.00	2.34
	2	4.76	1.93	0.77	1.71
	1	2.45	0.98	0.45	0.89
Base shear (kN)		1752.9	926.4	2459.7	1837.6

Table II. Response of the ER damped structure with instantaneouss control system.

	Storey number	El-Centro	Taft	Loma-Prieta	Eilat
Displacement (cm)	7	5.31	1.81	1.45	1.79
	6	5.14	1.72	1.42	1.77
	5	4.84	1.55	1.28	1.69
	4	3.76	1.35	1.09	1.55
	3	2.92	1.09	0.97	1.31
	2	2.20	0.84	0.82	1.02
	1	1.10	0.45	0.44	0.55
Base shear (kN)		1802.3	934.6	2470.3	1978.8

Table III. Response of the ER damped structure with predictive control system.

	Storey number	El-Centro	Taft	Loma-Prieta	Eilat
Displacement (cm)	7	5.36	1.85	1.58	1.85
	6	5.11	1.75	1.45	1.75
	5	4.60	1.56	1.26	1.61
	4	3.99	1.36	1.14	1.44
	3	3.20	1.09	1.04	1.18
	2	2.41	0.84	0.91	0.90
	1	1.27	0.45	0.50	0.48
Base shear (kN)		1766.8	935.1	2535.0	2166.7

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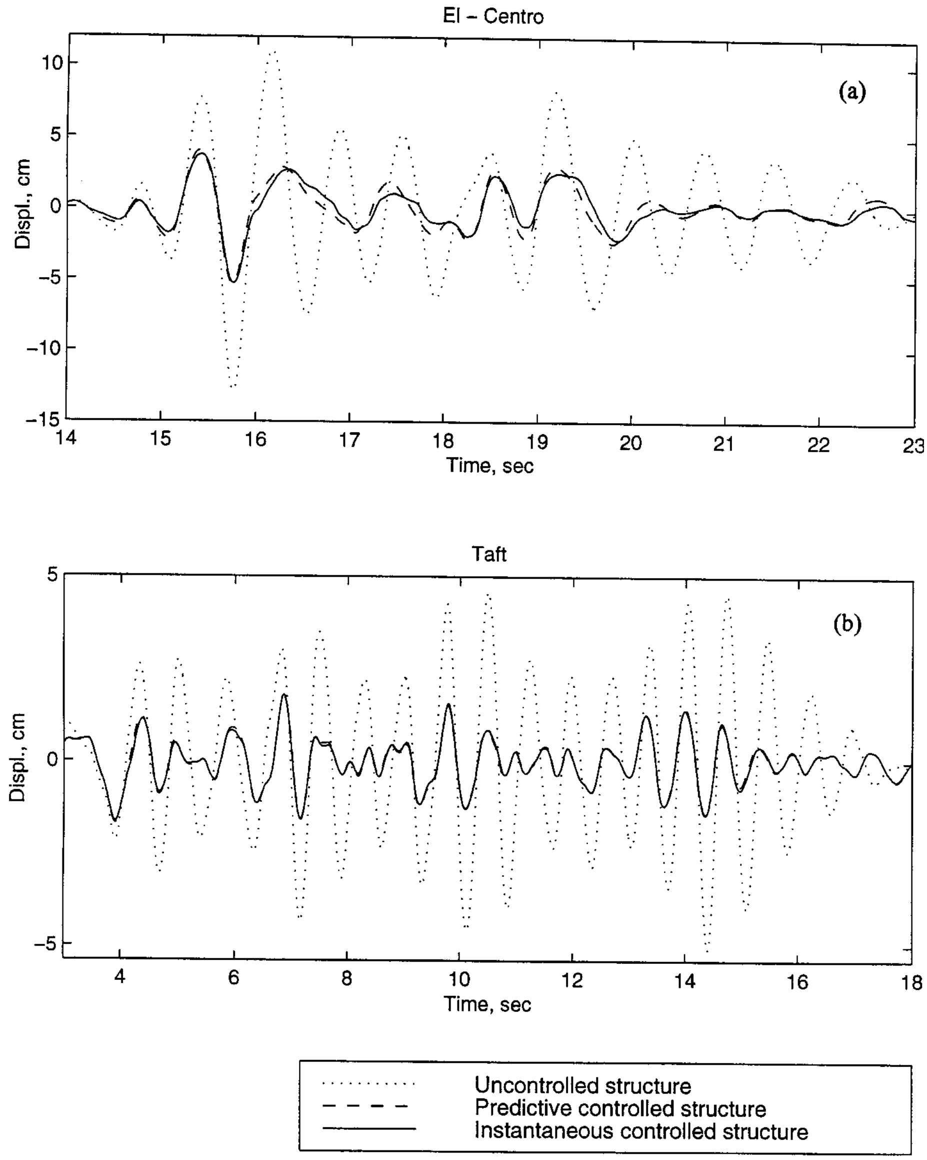

Figure 3. Roof displacement histories of the uncontrolled structure and of the active controlled structure with predictive and instantaneous system under: (a) El-Centro, (b) Taft, (c) Loma-Prieta, (d) Eilat earthquakes.

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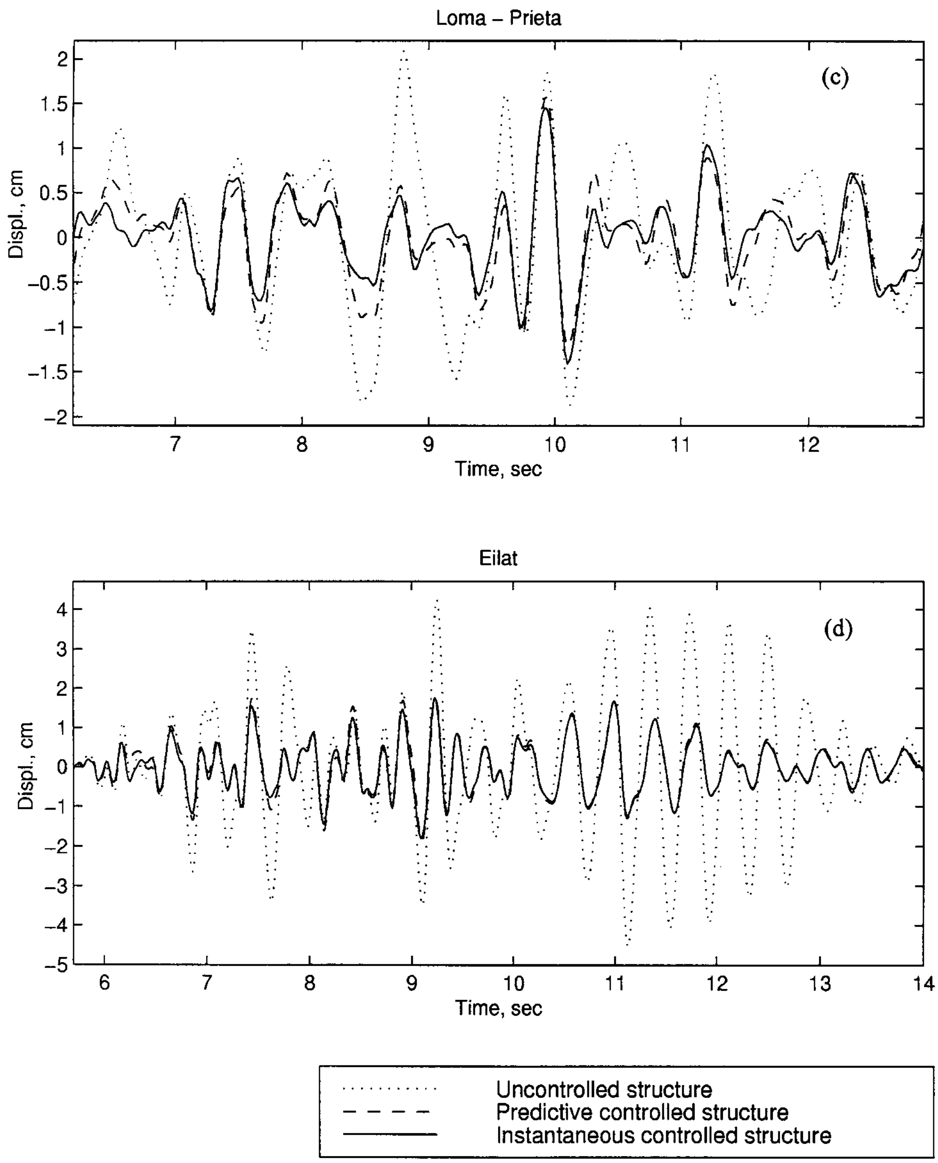

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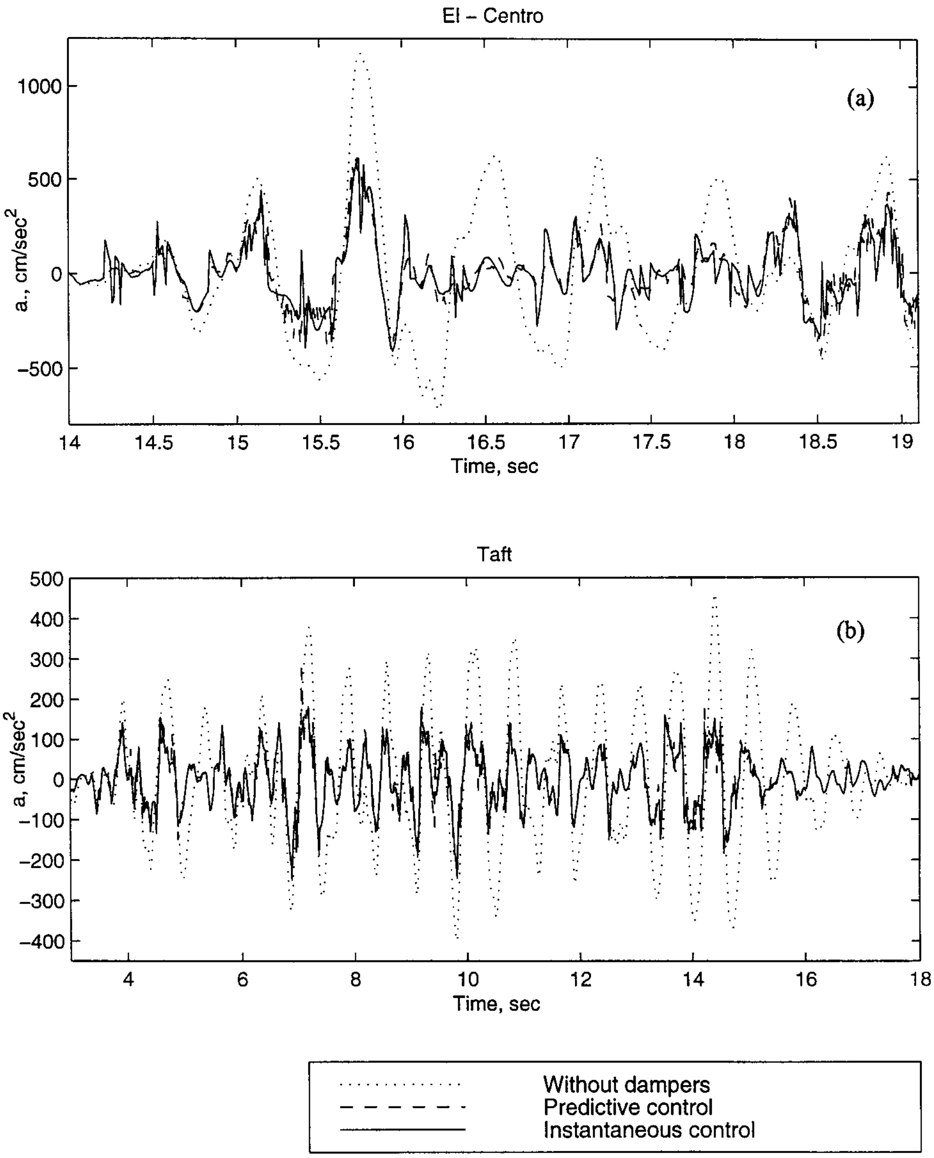
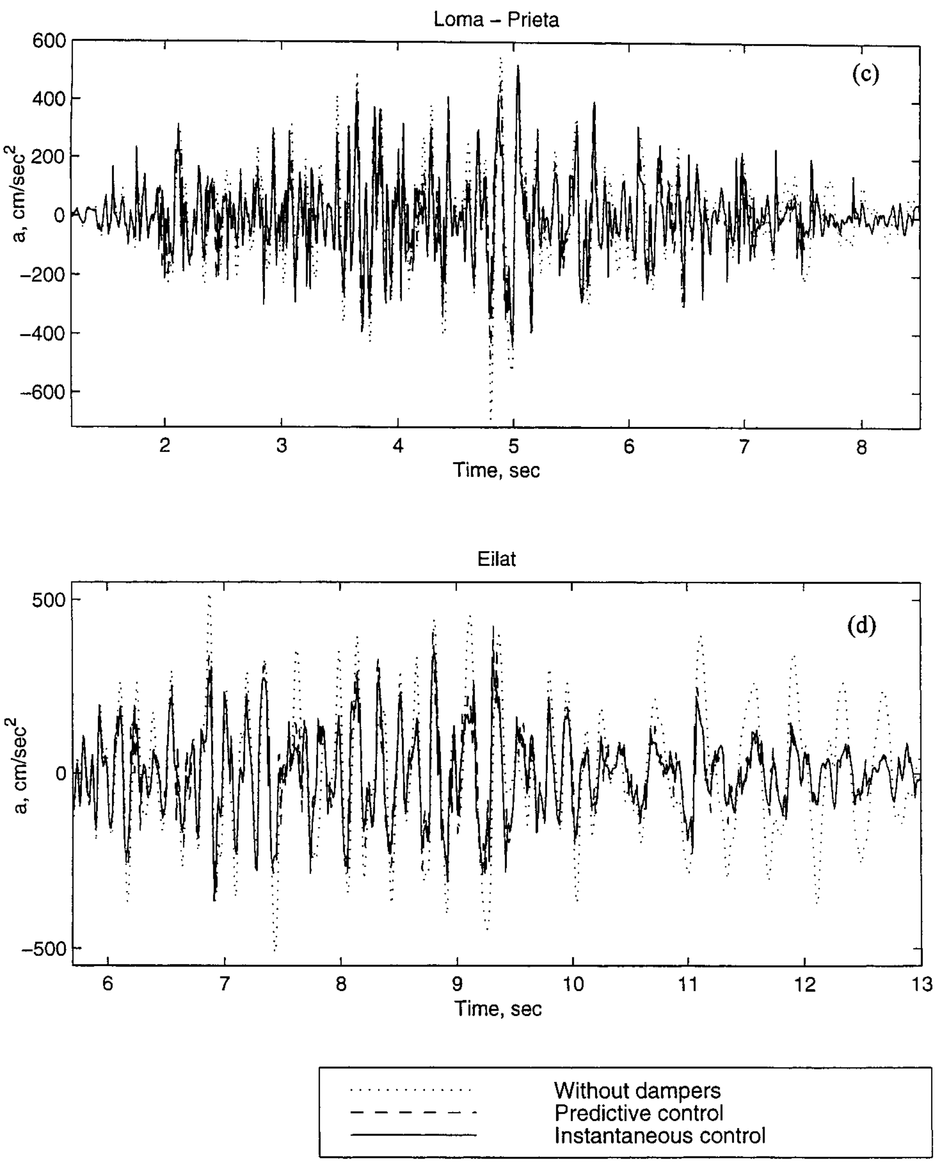


Figure 4. Roof acceleration histories of the uncontrolled structure and of the active controlled structure with predictive and instantaneous systems under: (a) El-Centro, (b) Taft, (c) Loma-Prieta (d) Eilat, earthquakes.



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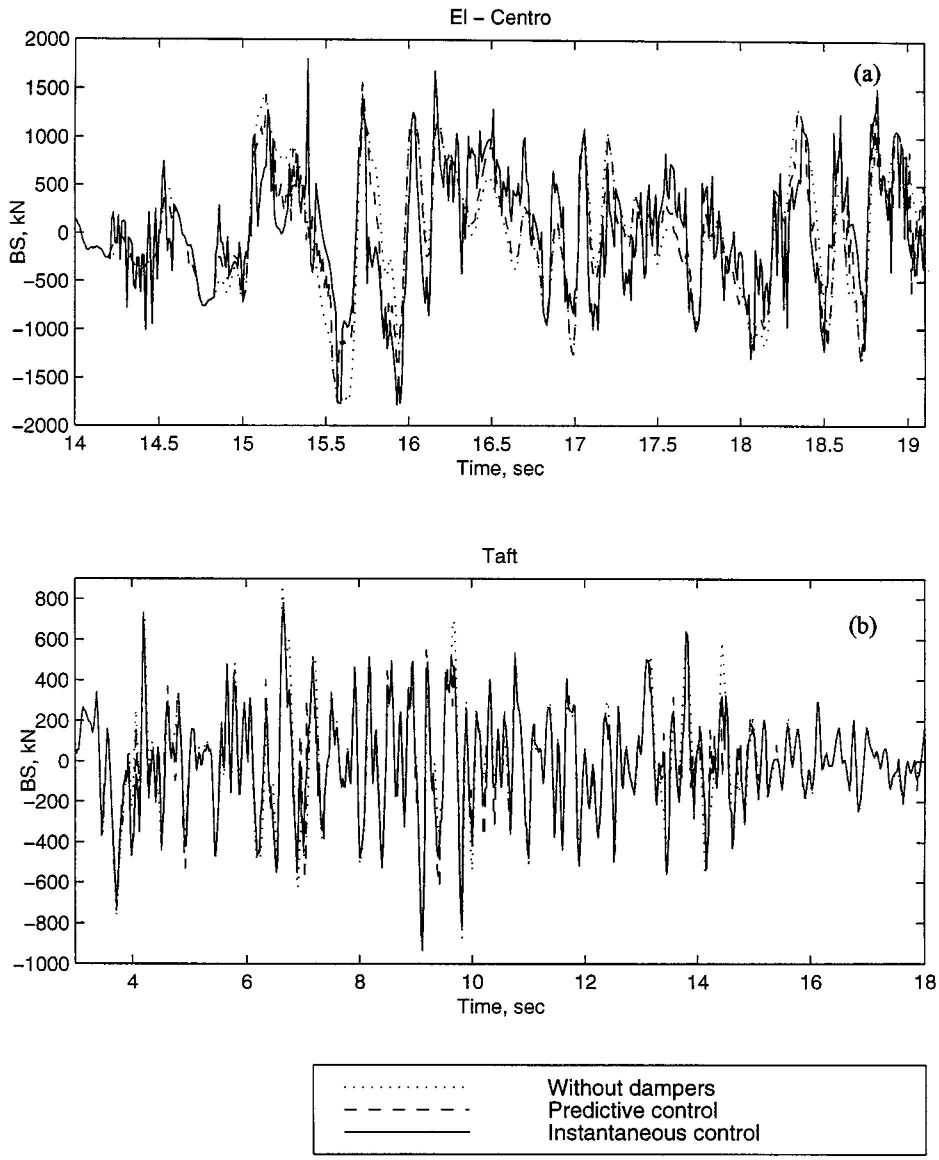

Figure 5. Base shear forces histories of the uncontrolled structure and of the active controlled structure with predictive and instantaneous system under: (a) El-Centro, (b) Taft, (c) Loma-Prieta, (d) Eilat earthquakes.

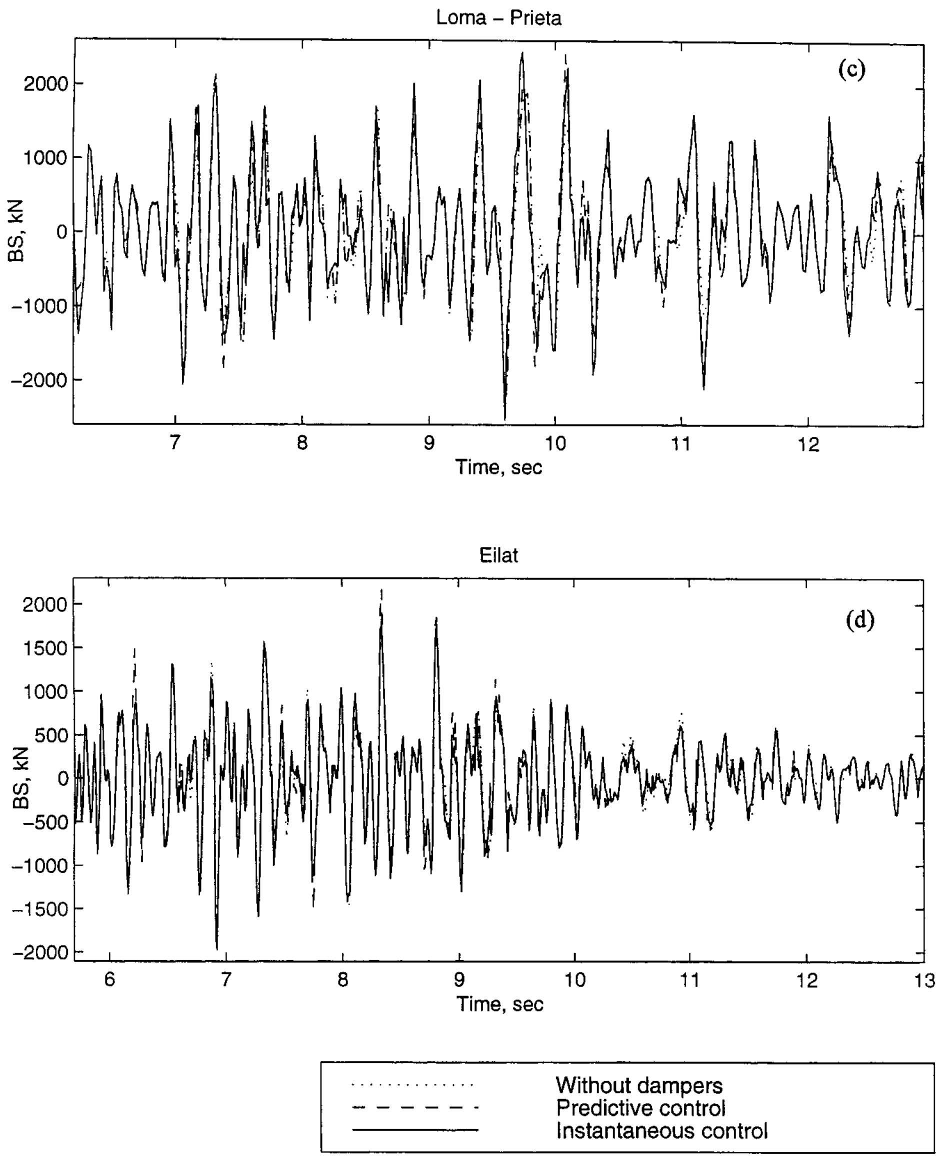

Figure 5. continued

CONCLUSIONS

A procedure was developed of a predictive control system for an optimal design of active controlled structures. This method was applied to ER damped structures. Viscous properties of the ER devices were selected using PCT, and ACT was then applied to find the optimal control forces by the proposed predictive control technique.

Thus, a linear ARX model can be used to predict at each instant of an occurring earthquake the structure's displacements and velocities at the next time step. The optimal control forces in the structure's ER dampers can be calculated during an earthquake and applied simultaneously with their corresponding displacements and velocities at the next time step (without delay). As a result, an improved behaviour of structures during earthquakes is achieved.

A numerical simulation of a seven-storey building was performed for the four following cases: an uncontrolled structure, a structure with an ideal instantaneous control system (without a time delay), a structure with a realistic control system with a time delay, and a structure with a predictive control system. The simulation shows reductions in the peak displacements and peak accelerations without a significant increase of the base shear forces. The structure's response with the predictive control system is similar to its response with an instantaneous one. The simulation also shows that at certain earthquake histories a structure with an active control system with a time delay may suffer significant damage, which can be prevented by the proposed predictive system.

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